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A CHOKED-FLOW CALCULATION CRITERION FOR NONHOMOGENEOUS, NONEQUILIBRIUM, TWO-PHASE FLOWS

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Abstract—This study applies the theory of characteristics to a one-dimensional transient model, in order to analyze the conditions for a choked, two-phase flow. The basic hydrodynamic model analyzed is a two-fluid model that includes relative phasic acceleration terms and a nonequilibrium, derivative–dependent exchange of mass.

The analytical results provide an algebraic, choked-flow criterion analogous to that for a single-phase flow, except that terms pertaining to relative phase motion and nonequilibrium mass transfer are included.

This paper discusses the numerical implementation of the choked-flow criterion in a nonhomogeneous and nonequilibrium finite difference scheme. The use of a mass-transfer model having a derivative dependence is shown to be necessary if self-choking is expected.

1. INTRODUCTION

The choked flow of a two-phase mixture is an important phenomenon in many two-phase flow situations. In particular, the choked, mass-discharge rate controls the depressurization rate (and the resulting energy inventory) in a light water reactor during blowdown conditions posited in a loss-of-coolant accident. In the past, homogenous or semiempirical models have been used to predict the choked, mass-discharge rate for use with system transient analytical models.

The present study developed a two-fluid, nonequilibrium, hydrodynamic model embodying additional degrees of freedom (in comparison with the homogeneous equilibrium model). This analysis developed an associated choked-flow criterion analogous to the well-known transient choked-flow criterion discussed in Shapiro (1954) for single-phase flows (that is, a criterion in which fluid velocity equals the local speed of sound). In two-phase flows, the speed of sound governing choked flow is much lower than the phasic sound speeds, but the exact value has been difficult to establish analytically, except for very special assumptions, such as the existence of a homogeneous, equilibrium flow.

The newly-developed choked-flow criterion is based on a two-fluid, analytical model for two-phase flow, and a single analytic expression was developed, relating the phasic veocities to a mixture sound-speed. The mixture sound-speed is a function of the interphase momentum coupling caused by relative acceleration (virtual mass) and the derivative–dependent, nonequilibrium mass exchange.

The analytical expression developed can be used to establish the choked, mass-flow rate as a function of local flow conditions. These relations are most useful in conjunction with numerical calculations of transient, two-phase flows. The use of a choked-flow criterion eliminates the need to model the flow process numerically in the immediate vicinity of the choked-flow point. Normally, large spatial gradients in the flow properties occur near points of choked flow and fine spatial noding is required for accurate resolution. The use of the choked-flow criterion and appropriate boundary conditions eliminates the need for such noding, which can be expensive in terms of computer storage and computation time.

Further, if self-choking in a numerical scheme is desired, it is shown that: (a) an appropriate derivative dependence is necessary in the interphase, mass-transfer rate; (b) an appropriate virtual-mass effect must be included in the interphase drag.

2. CHOKED-FLOW THEORY

Choked flow is defined as the condition wherein the mass-flow rate becomes independent of the downstream conditions (that is, that point at which further reduction in downstream pressure does not result in change of the mass-flow rate). Basically, a limit occurs because acoustic signals can no longer propagate upstream. This limit occurs when the fluid velocity just equals the propagation velocity.

For a differential operator, the path lines for a signal propagation are established from a characteristic analysis. A system of first-order, quasi-linear, partial differential equations of the form

$$A(U)[\partial U/\partial t] + B(U)[\partial U/\partial x] + C(U) = 0 \qquad [1]$$

is considered. The characteristic directions (or characteristic velocities) of the system are defined by Garabedian (1964) and Whitham (1974) as the roots, $\lambda_i (i \leq n)$,† of the characteristic equation

$$\det(A\lambda - B) = 0 . \qquad [2]$$

The eigenvalues of the characteristic equation are related to the general Fourier component of the solution for the locally linear system. The real part of any root λ_i^R gives the velocity of signal propagation along the corresponding characteristic path in the space-time plane. The imaginary part of any complex root λ_i^I gives the rate of growth or decay of the signal propagating along its respective path. For a hyperbolic system in which all the roots of [2] are real and nonzero, the number of boundary conditions required at any boundary point can be shown to equal the number of characteristic lines entering the solution region as t increases. If the system, [1], is applied in the particular spatial region $0 \leq x \leq L$ and the boundary conditions at $x = L$ are examined, it follows that as long as any λ_i is less than zero, some boundary information must be supplied in order to obtain the solution. If, on the other hand, all the λ_i are greater than or equal to zero, then no boundary conditions are needed at $x = L$ and the interior solution is unaffected by conditions beyond this boundary. A choked condition exists when no information can propagate into the solution region from the exterior. Such a condition exists at the boundary point $x = L$ when

$$\lambda_j = 0 \text{ for some } j \leq n \qquad [3]$$

$$\lambda_i \geq 0 \text{ for all } i \neq j . \qquad [4]$$

These are the mathematical conditions satisfied by the equations of motion for a flowing fluid when reduction in downstream pressure ceases to cause an increased flow rate. It is well-known (Shapiro 1954) that the choked-flow condition for single-phase flow occurs when the fluid velocity just equals the local sound speed.

Other possible critical flow criteria, some based on a steady state analysis, are found in the literature. For a complete discussion of the relationships that exist among these various criteria, the reader should consult Bouré *et al.* (1976).

3. A NONEQUILIBRIUM, MASS-EXCHANGE MODEL

The basic mass-transfer model proposed in this section assumes that the nonequilibrium, mass-exchange rate is, in a sense subsequently to be made precise, proportional to the

†Where n is the number of differential equations comprising the system defined by [1] and i designates any of the corresponding n roots.

equilibrium, mass-exchange rate. The equilibrium, mass-transfer rate is easily derived from the mixture-entropy equation for a reversible process,

$$\partial(\alpha_G\rho_G S_G + \alpha_L\rho_L S_L)/\partial t + \partial(\alpha_G\rho_G v_G S_G + \alpha_L\rho_L v_L S_L)/\partial x = 0\,. \qquad [5]$$

where α is the void fraction, ρ the phasic density, S the phasic entropy, v the phasic velocity and the subscripts G and L represent the gaseous and liquid phases respectively. The densities and entropies in [5] are functions of the pressure P only, evaluated along the respective saturation curves. Using the phasic continuity equations

$$\partial(\alpha_G\rho_G)/\partial t + \partial(\alpha_G\rho_G v_G)/\partial x = m_G{}^e \qquad [6]$$

$$\partial(\alpha_L\rho_L)/\partial t + \partial(\alpha_L\rho_L v_L)/\partial x = m_L{}^e \qquad [7]$$

and remembering that the mass excahnges $m_G{}^e$ and $m_L{}^e$ satisfy

$$m_L{}^e = -\,m_G{}^e\,,$$

we can solve the entropy equation for $m_G{}^e$ obtaining

$$m_G{}^e = \left[\alpha_G\rho_G S_G^*\left(\frac{\partial P}{\partial t} + v_G\frac{\partial P}{\partial x}\right) + \alpha_L\rho_L S_L^*\left(\frac{\partial P}{\partial t} + v_L\frac{\partial P}{\partial x}\right)\right]\bigg/(S_L - S_G)\,, \qquad [8]$$

where

$$S_G^* = \frac{\mathrm{d}S_G{}^s(P)}{\mathrm{d}P}\,,\quad S_L^* = \frac{\mathrm{d}S_L{}^s(P)}{DP}\,. \qquad [9]$$

Equation [8] gives the equilibrium, mass-exchange rate. Along with this mass exchange there must be a corresponding rate of heat transfer between the phases. To develop a consistent mass-exchange model this associated, reversible heat transfer must be known.

The reversible heat transfer between phases that gives rise to the equilibrium mass exchange can be found by considering the entropy equation for each phase undergoing a reversible process. In this situation we have

$$\partial(\alpha_G\rho_G S_G)/\partial t + \partial(\alpha_G\rho_G v_G S_G)/\partial x = m_G{}^e S_G + q_G{}^e/T_G \qquad [10]$$

$$\partial(\alpha_L\rho_L S_L)/\partial t + \partial(\alpha_L\rho_L v_L S_L)/\partial x = m_L{}^e S_L + q_L{}^e/T_L\,. \qquad [11]$$

where q is the heat transfer rate. The phasic temperatures, although equal for this case, have been denoted by T_G and T_L for later convenience. Equations [10] and [11], with $m_G{}^e$ and hence $m_L{}^e$ obtained from [8], can be solved for the equilibrium, heat-transfer rates† and give

$$q_G{}^3 = T_G\alpha_G\rho_G S_G^*\left(\frac{\partial P}{\partial t} + v_G\frac{\partial P}{\partial x}\right) \qquad [12]$$

$$q_L{}^e = T_L\alpha_L\rho_L S_L^*\left(\frac{\partial P}{\partial t} + v_L\frac{\partial P}{\partial x}\right)\,. \qquad [13]$$

†Bouré *et al.* (1981) has also recently noted that such a heat-transfer rate must be associated with the equilibrium mass transfer [8] to give a consistent theory.

To obtain the nonequilibrium, mass-exchange model it is assumed that

$$m_G = Km_G{}^e + m_G{}^n$$

$$q_G = Kq_G{}^e + q_G{}^n$$

$$q_L = Kq_L{}^e + q_L{}^n. \qquad [14]$$

K is a function of the state properties and $m_G{}^e$, $q_G{}^e$, $q_L{}^e$ are given by [8], [12] and [13] evaluated at the nonequilibrium state of the flowing mixture. $m_G{}^n$, $q_G{}^n$, $q_L{}^n$ are additional nonequilibrium exchange rates that are independent of any derivative terms. Because they are independent of derivatives, they need not be further specified in order to carry out the characteristic analysis—i.e., the $C(U)$ term in [1] has no effect upon the characteristic analysis of system [1].

It should be pointed out that the nonequilibrium mass-transfer model proposed above is strictly mathematical in nature. It does not relate the mass transfer to the local heat-transfer mechanism at the liquid–gas interface. Some surface renewal models for the interphase heat transfer have resulted in nonequilibrium mass-transfer formulations that are explicit functions of the pressure derivatives. Most nonequilibrium mass-transfer formulations are based upon the temperature difference between the bulk liquid or gas temperature and the saturation temperature assumed to exist at the interface. These relaxation-type models will not affect the critical-flow criterion formulated in section two, because they contain no derivative terms. To examine the critical flow phenomenon with such a mechanistic relaxation model for the mass transfer would require a detailed dispersion analysis with particular attention given to the dominant energy-carrying modes. Such an analysis is not conducted here; instead, we have formulated the nonequilibrium mass transfer as a constant fraction of the equilibrium mass transfer. The equilibrium formulation represents the limiting case obtained when the thermal resistance to heat flux is extremely small. Thus, the question of what effect the resistance has on the critical flow can be examined by variation of the constant K.

4. TWO-PHASE, CHOKED-FLOW ANALYSIS

The appropriate condition for choked flow of a two-phase fluid was developed by using the nonequilibrium, mass-transfer model of section 3. It is to be noted that two limiting cases are included in that model: (a) if $K = 1$, the thermal equilibrium case is obtained; (b) if $K = 0$, cases without mass exchange (frozen) are obtained. These cases bound actual two-phase flows in which thermal nonequilibrium exists.

The nondifferential source terms, $C(U)$, in [1] do not enter into the characteristic analysis, and thus do not affect the propagation velocities. For this reason the source terms associated with wall friction, interphase drag, and heat transfer are omitted for brevity in the following system of equations. The two-fluid model is described by a system of equations that includes: the two phasic mass-continuity equations; the two phasic momentum equations; and the two phasic entropy equations. This system is

$$\partial(\alpha_G\rho_G)/\partial t + \partial(\alpha_G\rho_G v_G)/\partial x = m_G \qquad [15]$$

$$\partial(\alpha_L\rho_L)/\partial t + \partial(\alpha_L\rho_L v_L)/\partial x = m_L \qquad [16]$$

$$\begin{aligned} &\alpha_G\rho_G[\partial v_G/\partial t + v_G(\partial v_G/\partial x)] + \alpha_G(\partial P/\partial x) \\ &+ C\alpha_G\alpha_L\rho[\partial v_G/\partial t + v_L(\partial v_G/\partial x) - \partial v_L/\partial t - v_G(\partial v_L/\partial x)] \\ &-(v_{IG} - v_G)m_G = 0 \end{aligned} \qquad [17]$$

$$\alpha_L\rho_L[\partial v_L/\partial t + v_L(\partial v_L/\partial x)] + \alpha_L(\partial P/\partial x)$$
$$+ C\alpha_L\alpha_g\rho[\partial v_L/\partial t + v_g(\partial v_L/\partial x) - \partial v_G/\partial t - v_L(\partial v_G/\partial x)]$$
$$-(v_{IL} - v_L)m_L = 0 \qquad [18]$$

$$\partial(\alpha_G\rho_G S_G)/\partial t + \partial(\alpha_G\rho_G S_G v_G)/\partial x - m_G S_G - q_G/T_G = 0 \qquad [19]$$

$$\partial(\alpha_L\alpha_L S_L)/\partial t + \partial(\alpha_L\rho_L S_L v_L)/\partial x - m_L S_L - q_L/T_L = 0 \qquad [20]$$

where v_{IL} and v_{IG} represent the average interphase velocity for momentum transfer caused by mass transfer and C is an added mass constant.

The momentum equations include the model for interphase force terms caused by relative acceleration discussed in Lahey (1977). These force terms have a significant effect on wave propagation velocity and, consequently, on the choked-flow velocity. The particular form chosen is frame invarient and symmetric. The coefficient of virtual mass, $C\alpha_G\alpha_L\rho$, is chosen to assure a smooth transition between pure vapor and pure liquid. For a dispersed flow, the constant (C) has a theoretical value of 0.5, whereas, for a separated flow, the value may approach zero.

Before carrying out the characteristic analysis of [15]–[20], the entropy equations will be examined in more detail. When the constitutive relationship [14] are substituted into [19] and [20], and all nondifferential terms neglected, one obtains

$$\frac{\partial S_G}{\partial t} + v_G\frac{\partial S_G}{\partial x} - KS_G^*\left(\frac{\partial P}{\partial t} + v_G\frac{\partial P}{\partial x}\right) = 0 \qquad [21]$$

$$\frac{\partial S_L}{\partial t} + v_L\frac{\partial S_L}{\partial x} - KS_L^*\left(\frac{\partial P}{\partial t} + v_L\frac{\partial P}{\partial x}\right) = 0 \qquad [22]$$

as the phasic entropy equations. These equations, along with [15] through [18], form the basic set of equations which must be analyzed to establish the choked-flow criterion.

When the state equations

$$\rho_G = \rho_G(P, S_G), \quad \rho_L = \rho_L(P, S_L)$$

are used, the system of governing equations can be written in terms of the six dependent variables, α_G, P, v_G, v_L, S_G, and S_L.

Thus, the system of equations can be written in the form of [1], where A and B are sixth-order, square coefficient matrices. The characteristic determinant corresponding to this system, [2], yields the following sixth-order polynominal in λ:

$$(\lambda - v_G)(\lambda - v_L)(\rho C(\lambda - v_L)(\lambda - v_G) + \alpha_L\rho_G(\lambda - v_G)^2 + \alpha_G\rho_L(\lambda - v_L)^2$$
$$-\{[\rho_G(\lambda - v_G) - \rho_L(\lambda - v_L)][E_G(\lambda - v_G) + E_L(\lambda - v_L)]$$
$$+(\alpha_L\rho_G I_L + \alpha_G\rho_L I_G)(\lambda - v_L)(\lambda - v_G)\}[(\lambda - v_L)(\lambda - v_G)$$
$$+(C\rho\alpha_L/\rho_G)(\lambda - v_L)^2 + (C\rho\alpha_G/\rho_L)(\lambda - v_G)^2]$$
$$+[E_G(\lambda - v_G) + E_L(\lambda - v_L)][C\rho(v_L - v_G)(\lambda - v_G)(\lambda - v_L)$$
$$+\rho_L(\lambda - v_L)^2(v_{IG} - v_G) - \rho_G(\lambda - v_G)^2(v_{IL} - v_L)]) = 0. \qquad [23]$$

where

$$I_G = \left.\frac{\partial \rho_G}{\partial P}\right|_{S_G} + K\left.\frac{\partial \rho_G}{\partial S_G}\right|_P S_G^*$$

$$I_L = \left.\frac{\partial \rho i_L}{\partial P}\right|_{S_L} + K\left.\frac{\partial \rho_L}{\partial S_L}\right|_P S_G^*$$

$$E_G = K\alpha_G\rho_G S_G^*/(S_L - S_G)$$

$$E_L = K\alpha_L p_L S_L^*/(S_L - S_G). \qquad [24]$$

Two roots of [23] are easily seen to be

$$\lambda_1 = v_G$$

$$\lambda_2 = v_L. \qquad [25]$$

These roots are a direct result of the entropy equations [21] and [22]. They were already in characteristic form and show the entropy changes associated with the pressure changes along their respective characteristic directions.

The remaining fourth-order polynomial in [23] can be factored approximately to obtain the remaining roots for λ, and thus can establish the choked-flow criterion. The factorization is presented in appendix A and produces the following results.† The approximate expressions for the first two roots are

$$\begin{aligned}\lambda_{3,4} = {} & [\{\alpha_L\rho_G + \rho C/2 \pm [(\rho C/2)^2 - \alpha_G\alpha_L\rho_G\rho_L]^{1/2}\} v_G \\ & + \{\alpha_G\rho_L \mp \rho C/2 \mp [(\rho C/2)^2 - \alpha_G\alpha_L\rho_G\rho_L]^{1/2}\} v_L] \\ & /((\alpha_L\rho_G + \rho C/2) + (\alpha_G\rho_L + \rho C/2)). \end{aligned} \qquad [26]$$

The values for $\lambda_{3,4}$ may be real or complex, depending on the sign of the quantity $(\rho C/2)^2 - \alpha_G\alpha_L\rho_G\rho_L$.

The remaining roots are:

$$\lambda_{5,6} = v + D(v_G - v_L) \pm a \qquad [27]$$

where

$$v = (\alpha_G\rho_G v_G + \alpha_L\rho_L v_L)/\rho,$$

$$a = a_H\{[C\rho^2 + \rho(\alpha_G\rho_L + \alpha_L\rho_G)]/(C\rho^2 + \rho_G\rho_L)\}^{1/2}, \qquad [28]$$

$$\begin{aligned}\alpha_H^{\,2} = (\rho_G\rho_L/\rho)\Big[& \alpha_L\rho_G\left(\frac{\partial \rho_L}{\partial P} + K\frac{\partial \rho_L}{\partial S_L}S_L^*\right) + \alpha_G\rho_L\left(\frac{\partial \rho_G}{\partial P} + K\frac{\partial \rho_G}{\partial S_G}S_G^*\right) \\ & + (\rho_G - \rho_L)K(\alpha_G\rho_G S_G^* + \alpha_L\rho_L S_L^*)/(S_L - S_G)\Big]^{-1}, \end{aligned} \qquad [29]$$

and the complicated analytic expression for D is given in appendix A.

The general nature and significance of these roots is revealed by applying the characteristic considerations discussed in section 2. The speeds with which small disturbances propagate are related to the values of the characteristic roots. In general, the velocity of propagation

†In this factorization we used $v_{IG} = v_{IL} = v_I$, with $v_I = 1/2(v_G + v_L)$—(Wallis 1969). A second assumption for v_I, $v_I = v_L$ if $m_G > 0$ and $v_I = v_G$ if $m_G < 0$, which always makes the mass exchange process dissipative, was also used with no significant change in the plots for the roots.

corresponds to the real part of a root, and the growth or attenuation is associated with the complex part of a root. The choked-flow condition concerns the velocity with which a disturbance propagates at a point fixed in space. Thus, the choked-flow criterion is established from examination of the real part of a characteristic root. A choked condition will exist when the signal, which propagates with the largest velocity relative to the fluid, is just stationary; that is,

$$\lambda_j^R = 0 \text{ for some } j \leq 6 \tag{30}$$

and

$$\lambda_i^R \geq 0 \text{ for all } i \neq j. \tag{31}$$

The existence of complex roots for $\lambda_{3,4}$ makes the initial-boundary value problem ill-posed. This problem has been discussed by many investigators, and it is only noted here that the addition of any small, second-order, viscous effect renders the problem well-posed (Jackson 1970). The phenomenon of systems with mixed orders of derivatives and, in particular, of a first-order system with the addition of a small second-order term, has been discussed and analyzed by Whitham (1974). He has shown that the second-order viscous terms do give infinite propagation velocities, but that the bulk of the information is propagated along the characteristic lines defined by the first-order system. It is concluded that the ill-posed nature of [15]–[20] can be removed by the addition of small, second-order, viscous terms and that these terms will have little effect upon the propagation of information. Therefore, the choked-flow criterion for the two-phase flow system analyzed here is established from [30].

The character of the choked-flow criterion for the two-phase flow model defined by [15]–[20] will now be examined. Since the real parts of the two roots $\lambda_{3,4}$ are between the phase velocities v_L and v_G, the choked-flow criterion is established from the roots $\lambda_{5,6}$ and [30]. The choked criterion is

$$v + D(v_G - v_L) = \pm a. \tag{32}$$

This criterion can be rewritten in terms of the mass mean and relative Mach numbers

$$M_v = v/a, \ M_r = (v_G - v_L)/a \tag{33}$$

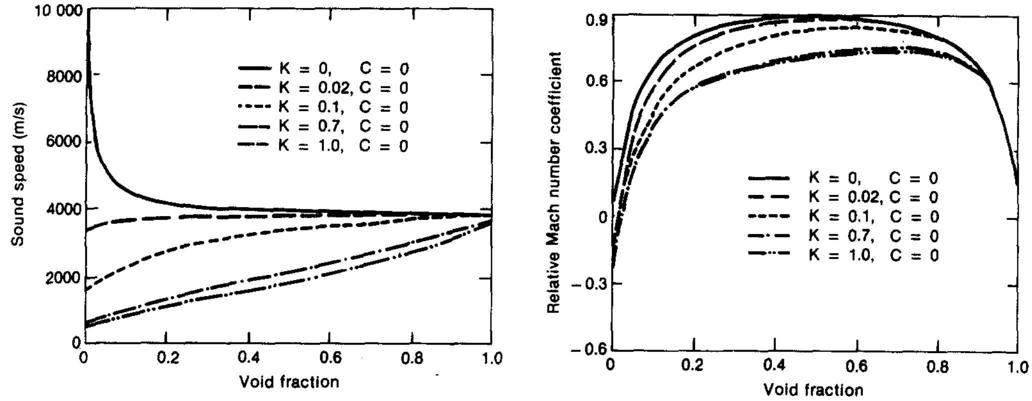
as

$$M_v + DM_r = \pm 1. \tag{34}$$

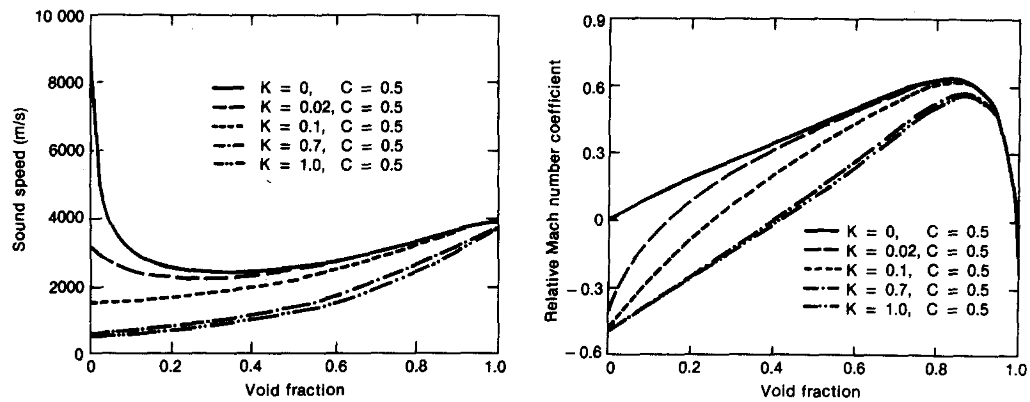
This relation is very similar to the choked-flow criterion for a single-phase flow.

The choked-flow criterion, [34], is a function of parameters D and a. In figures 1–5, a is plotted as a function of the void fraction α_G for a typical steam-water system at 7.5 MPa. The limiting values, $K = 1$, $C = \infty$, correspond to the (known) homogeneous, equilibrium case, whereas the other limiting values, $K = 0$, $C = 0$, correspond to the (known) stratified frozen case. The first case could be expected in a tightly-coupled, dispersed-flow situation and the second in an annular separated flow of two components.

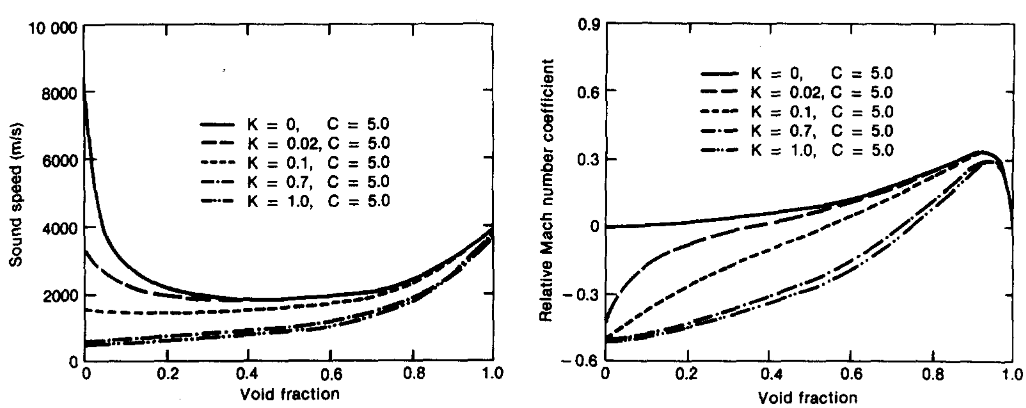
Figures 1–5 show that the virtual mass coefficient has a significant effect upon choked-flow models for two-phase flows (Anderson *et al.* 1977), as does the nonequilibrium, derivative-dependent mass exchange. To establish the actual choked-flow rate for nonhomogeneous, two-phase flow, the relative velocity term in [34] must be considered. The relative Mach number coefficient, D, is plotted in figures 1–5. These results show that the choked-flow velocity criterion can differ appreciably from the mass mean velocity when slip occurs.



(a) Fig. 1. (b)



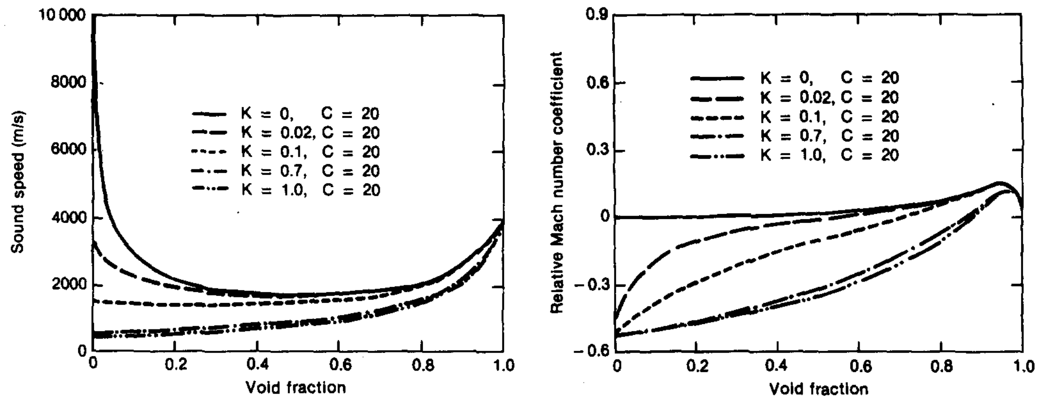
(a) Fig. 2. (b)



(a) Fig. 3. (b)

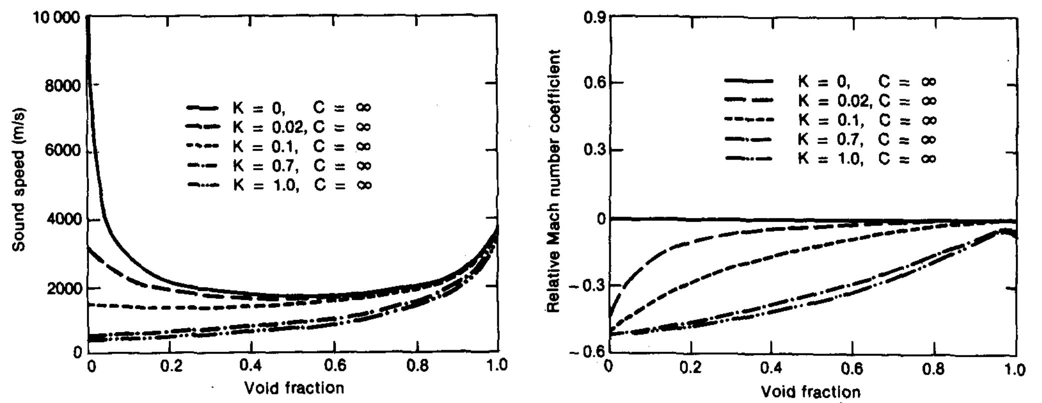
Several observations can be made regarding figures 1–5. First, note that both the inertial coupling terms and the derivative-dependent mass exchange are responsible for a significant depression of the sound speed, a, and hence for greatly reduced critical flow rates—when compared to models that neglect these terms.

Also note that the limiting sound speeds on the $K = 0$, $C = 0$ curves at $\alpha_G = 0$ and 1 are the single-phase liquid and gas sound speeds, respectively. The discontinuities that exist at $\alpha_G = 0$ and 1 for all K curves, except $K = 0$, are caused by the assumption of a non-zero, mass-transfer rate which gives a "bulk" discontinuity in the density/pressure relationships at $\alpha_G = 0$ and 1. This is



(a) (b)

Fig. 4.



(a) (b)

Fig. 5.

most significant on the liquid side, where any mass exchange makes the mixture "spongy", relative to the stiff liquid response.

Finally, note from the figures that the variation in the relative Mach-number coefficient is of the order unity and hence the velocity that is set equal to the sound speed in [32] can, as α_G varies, range from near v_G to near v_L. Several investigators have found limiting cases for the sound speed, a, but the relative velocity effect has not been obtained from the basic equations before. Because of the large difference in density for most two-phase flows, the depressurization rate (controlled by the critical-flow criterion) is very sensitive to the value of D. Different values of D indicate different proportions of liquid or gas leaving the system. The choked-flow criterion was used in the RELAP5 code† to model the choked-discharge flow in the pipe blowdown experiment performed by Edwards & O'Brien (1970). The model was used to provide boundary conditions for the RELAP5 transient, nonequilibrium, numerical model for system-hydrodynamic calculations. The results obtained from the blowdown calculations did indeed show dramatic changes in the depressurization rate as K and C were varied.

4. A CHOKED-FLOW CRITERION FOR TWO-PHASE FLOWS

The criterion which is satisfied for a choked flow is established from [34] with the consideration that either right or left traveling acoustic waves may satisfy the identity, depending upon the sign of

†The RELAP5/MOD0 code and associated documentation (V. H. Ransom *et al.*, *RELAP5/MOD0 Code Description*, Vols. 1–3, CDAP-TR-057, May 1979) are available at the National Energy Software Center, Building 208, Room C-230, 9700 South Cass Avenue, Argonne, IL 60439, U.S.A.

the flow velocity. The criterion is

$$|W_v + DM_r| = 1.0\,. \qquad [35]$$

When a flow is detected to be choked or is specified to be choked, [35] is used as a boundary condition for the flow solution. The relative velocity, in addition to the mass average velocity, enters into the choked-flow criterion so that [35] must be solved simultaneously with the equations of motion. The relative velocity ($v_G - v_L$) and the degree of thermal nonequilibrium which exist in a flow approaching a point of choked flow will affect the choked-flow rate.

4.1 *Development of the choked-flow criterion*

The conditions and/or parameters which enter into the choked-flow criterion include all of the flow variables, as well as the virtual mass coefficient, C, and the nonequilibrium, mass-transfer parameter, K. The flow variables must be established from the solution of the equations of motion for the flow preceding the critical flow point (Giot & Fritte 1971). The virtual-mass coefficient was established from analytical considerations (Zuber 1964). For highly dispersed bubbly and droplet flows, the value of the virtual-mass coefficients is approximately 0.5, and for separated flows, the value approaches zero. If interactions between the dispersed phase bubbles or drops are included, values for the virtual mass coefficient have been estimated to be as large as four (Zuber 1964). Thus, the coefficient is known within a range.

The degree of thermal nonequilibrium (i.e. the value of K) which exists in a two-phase flow has a large effect on the choked-flow rate. This can be seen in figures 1–5. The degree of nonequilibrium which should be included in the choked-flow analysis must be investigated empirically at the present time by comparisons to measured data.

5. CONCLUSIONS AND RECOMMENDATIONS

A theoretically-based and computationally-efficient model for calculating the choked-mass discharge of a two-phase mixture has been developed. The model is independent of a finite differencing scheme and can be applied to flow at a sharp edged orifice where fine spatial differencing becomes impractical.

In fluid-flow problems which must be solved using finite difference numerical schemes, it is very tempting to assume that the calculations will correctly predict the choked-flow rates for sufficiently small mesh spacing and time step. This would in fact be the case if the differential equations correctly describe the physics of the flow process and if consistent difference operators are used. In the case of two-phase flow, however, the correct form for the differential equations is the subject of considerable debate (Gidaspow 1974). As example of this controversy, the virtual-mass terms are debated, both about whether they should be included and about their proper form. These terms have a large effect on the choked-flow rate, as has been shown herein. A numerical calculation in which these terms are omitted would yield results approximating the choked-flow analysis for a value of the virtual mass coefficient, C, equal to zero. The corresponding difference in choked-flow velocity is illustrated in figures 1–5.

The magnitude of the virtual-mass term depends upon the spatial and temporal derivatives, and is a small part of the interphase drag for situations in which long wave-length phenomena predominate, such as steady flow in a smooth duct. However, when short wave-length phenomena are important, such as at a point of choked flow, then the virtual-mass terms can predominate. Hence, it may be possible to neglect these terms in a nonchoked-flow calculation, but it is essential that they be included in the numerical scheme for a region of choked flow.

Similar remarks can be made concerning the modeling used for the mass transfer in a two-phase system. The sound speed for $K = 0$ is for a system having a mass-transfer model that failed to include differential terms, while the sound speeds shown for $K \neq 0$ are for systems with differential mass-transfer terms. These derivative terms account for a large reduction in the choked-flow velocity. The experimental data from Wallis (1969) show a choked-flow velocity very close to the equilibrium model predictions. Here again, any numerical scheme used to predict the critical-flow rate directly must contain a mass-transfer model that includes derivative terms similar to those of the equilibrium model.

A further note relative to numerical, choked-flow calculations concerns the fact that any such calculations for finite spatial and temporal increments are only an approximation of the original differential system. As a result of truncation errors, the numerical solution is actually the solution to an augmented differential system. The additional terms are equivalent to first-order derivatives having coefficients containing the spatial and temporal increments. Hence, for finite increments, the numerical solution for choked flow will correspond to a system having additional first-order derivatives present. In order to correctly calculate choked-flow directly, it is important to ensure that these truncation terms are small, relative to the correct, first-order, derivative terms in the basic differential equations. This convergence can be achieved in practice by choosing successively smaller computing increments (time and space), so that the calculated choked flow becomes constant.

The alternative approach to direct, numerical, choked-flow calculation is the use of a choked-flow criterion such as the approach developed herein. This approach has been used extensively in the case of single-phase flows. In general, the choked-flow criterion, [34], is used both as a criterion to determine if a flow is choked, and as a boundary condition when the flow is choked. The advantage of this approach is that the calculation is less sensitive to the computing interval, so that large spatial increments can be used with attendant savings in computational time. In addition, one is assured that the correct choked-flow velocity is specified, independently of the finite differencing algorithm.

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6. REFERENCES

ANDERSON, P. S., ASTRUP, P. P., EGET, L. & RATHMANN, O. 1977 Numerical experience with the two-fluid model, RISQUE, *Proc. ANS Water Reactor Safety Meeting.* 31 July–4 August.

BOURÉ, J. A., FRITTE, A. A., GIOT, M. M. & RÉOCREUX, M. L. 1976 Highlights of two-phase critical flow: on the links between maximum flow rates, sonic velocities, propagation and transfer phenomena in single and two-phase flows. *Int. J. Multiphase Flow* **3**, 1–22.

BOURÉ, J. A. 1981 The closure laws for one-dimensional two-phase flow models: propagation phenomena and restrictions to algebraic and first-order partial derivative forms. *3rd CSNI Specialist Meeting on Transient Two-Phase Flow.* Pasadena.

EDWARDS, A. R. & O'BRIEN, T. P. 1970 Studies of phenomena connected with the depressurization of water reactors. *J. Br. Nuclear Energy Soc.* **9**, 128–135.

GARABEDIAN, P. R. 1964 *Partial Differential Equations.* McGraw-Hill, New York.

GIDASPOW, D. 1974 Modeling of two-phase flow modeling program. *Proc. Round Table Discussion RT-1-2 at the Fifth Int. Heat Transfer Conf.* Tokyo, Japan, 3–7 September.

GIOT, M. & FRITTE, A. 1971 Two-phase two- and one-component critical flows with the variable slip model. *Prog. Heat Mass Transfer* **6**, 651–670.

JACKSON, R. 1970 The present status of fluid mechanical theories of fluidization. *Chem. Engng Prog. Symp. Ser.* **66**, 3–13.

LAHEY, R. T. JR. 1977 RPI two-phase flow modeling program. *NRC Fifth Water Reactor Safety Research Meeting.* Gaithersburg, Maryland, 7 November.

SHAPIRO, A. H. 1954 *The Dynamics and Thermodynamics of Compressible Fluid Flow*, Vol. II. Ronald Press, New York.

WALLIS, G. B. 1969 *One-Dimensional Two-Phase Flow.* McGraw-Hill, New York

WHITHAM, G. B. 1974 *Linear and Nonlinear Waves.* Wiley, New York.

ZUBER, N. 1964 On the dispersed two-phase flow in the laminar flow region. *Chem. Engng Sci.* **19**, 897–917.

APPENDIX A

Factorization of characteristic polynomial

The fourth-order polynomial obtained as the solution to the characteristic analysis must be factored in order to obtain the choked-flow criterion. This can only be carried out approximately for the general case of unequal phase velocities. However, some useful insight can be obtained by first considering the case of equal phase velocities (that is, $v_G = v_L = v_0$). For this case, [23] of the main text can be factored exactly, with the following results:

$$\lambda_{3,4} = v_0 \tag{A1}$$

$$\lambda_{5,6} = v_0 \pm a \tag{A2}$$

where the sound speed, a, is defined as before by [28]. Hence, for small values of $v_G - v_L$, two of the roots are expected to be near the phasic mixture velocity, v, and thus the factors $\lambda - v_G$ and $\lambda - v_L$ will be of the order $v_G - v_L$. This being the case, it is possible to obtain a close approximation for these two roots by neglecting the fourth-order factors in $\lambda - v_G$ and $\lambda - v_L$ relative to the second-order factors. The remaining two roots are expected to be of the order $v \pm a$, and the factors $\lambda - v_G$ and $\lambda - v_L$ to be of the order $\pm a$ (that is, not small). The roots $\lambda_{3,4}$ may be interpreted physically as the paths along which kinematic effects propagate at the fluid velocity, while the roots $\lambda_{5,6}$ are the paths along which acoustic phenomena propagate at speeds $v \pm a$.

Using the above information, the approximate factorization of the fourth-order polynomial, [23] of the text, may now be described. Since the slower kinematic roots (corresponding to [A1]) are near v_L and v_G, the factors $\lambda - v_G$ and $\lambda - v_L$ are small for these roots. Therefore, when approximating these roots, the fourth-order factors in $\lambda - v_L$, and $\lambda - v_G$ relative to the second-order factors in $\lambda - v_G$ and $\lambda - v_L$ can be neglected. This produces the following quadratic polynomial for the kinematic roots:

$$\rho C(\lambda - v_G)(\lambda - v_L) + \alpha_L\rho_G(\lambda - v_G)^2 + \alpha_G\rho_L(\lambda - v_L)^2 = 0 \tag{A3}$$

with corresponding solutions as given in the text by [26].

Note that these roots do in fact lie between v_L and v_G, thus confirming the assumption that the factors $\lambda - v_G$ and $\lambda - v_L$ are small.

If these were exact roots of [23], then [23] could be divided by them to obtain an exact quadratic polynomial that would lead directly to the remaining acoustic roots. Instead, division using the approximate roots is performed. The resulting polynomial is then approximated by using the identities

$$v_G = v + \frac{\alpha_L\rho_L}{\rho}(v_G - v_L) \tag{A4}$$

$$v_L = v - \frac{\alpha_G\rho_G}{\rho}(v_G - v_L) \tag{A5}$$

and by neglecting all the terms in $v_G - v_L$ that are second-order and higher. This leads to a quadratic containing, at most, terms linear in $v_G - v_L$ and which can be factored exactly to yield approximations for the remaining two roots of [23]. The roots are

$$\lambda_{5,6} = v + D(v_G - v_L) \pm a\dagger \tag{A6}$$

†Assuming that the relative velocity, $v_G - v_L$, is not large.

where v, a, and a_H are as defined by [27]–[29] and the complicated analytic expression for D is defined by

$$D = \frac{-\sum_{i=1}^{4} Y_i + \sum_{i=1}^{5} X_i Z_i - B/a^2}{2\sum_{i=1}^{5} X_i} \qquad \text{[A7]}$$

where

$$B = C(\alpha_L\rho_L - \alpha_G\rho_G) + 2\rho_L\rho_G(\alpha_L{}^2 - \alpha_G^2)/\rho$$

$$X_1 = \rho_G E_G C\rho\alpha_G/\rho_L$$

$$X_2 = \rho_G E_G + AC\rho\alpha_G/\rho_L$$

$$X_3 = A + \rho_G E_G C\rho\alpha_L/\rho_G - \rho_L E_L C\rho\alpha_G/\rho_L$$

$$X_4 = AC\rho\alpha_L/\rho_G - \rho_L E_L$$

$$X_5 = -\rho_L E_L C\rho\alpha_L/\rho_G$$

$$A = (\rho_G E_L - \rho_L E_G) + \alpha_L\rho_G I_L + \alpha_G\rho_L I_G$$

$$Y_1 = 0.5\rho_G E_G$$

$$Y_2 = C\rho E_G + 0.5\rho_G E_L$$

$$Y_3 = C\rho E_L + 0.5\rho_L E_G$$

$$Y_4 = 0.5\rho_L E_L$$

$$Z_1 = 4\alpha_L\rho_L/\rho$$

$$Z_2 = 3\alpha_L\rho_L/\rho - \alpha_G\rho_G/\rho$$

$$Z_3 = 2\alpha_L\rho_L/\rho - 2\alpha_G\rho_G/\rho$$

$$Z_4 = \alpha_L\rho_L/\rho - 3\alpha_G\rho_G/\rho$$

$$Z_5 = -4\alpha_G\rho_G/\rho$$

and the terms E_G, E_L, I_G and I_L are defined by [24].

In the formula for D we have assumed that the interface velocity, v_I, was equal to $1/2\,(v_L + v_G)$.